

Helicopter Rotor Lift Distributions for Minimum-Induced Power Loss

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A method is described for solving the minimum-induced loss (MIL) rotor design problem. First, the generalized Betz condition for MIL rotors is developed. Because the resulting lift distributions would generally exceed the maximum blade lift coefficient on the retreating side of the rotor, the necessary conditions are extended to include constraints on the lift. A method for solving for the optimum lift distribution using finite elements is described. Numerical results are presented for a typical rotor in forward flight. The MIL rotor may have on the order of 10% less induced power loss than a typical unoptimized rotor.

Nomenclature

A = rotor disk area, πR^2
 a = blade section lift curve slope
 B = matrix associated with force constraints
 C = matrix associated with moment constraints
 C_L = section lift coefficient
 C_P = coefficient of power, $P/\rho A(\Omega R)^3$
 C_T = thrust coefficient, $T/\rho A(\Omega R)^2$
 c = blade chord
 E = kinetic energy in wake
 F = rotor force vector
 G = matrix relating circulation to the potential
 H = aft in-plane rotor force
 $\hat{i}, \hat{j}, \hat{k}$ = unit vectors in the aft, right, up directions
 K = finite element matrix
 L = lift produced by a single blade
 M = vector of rotor moments
 M_r = rotor hub rolling moment
 $M_{r\dot{\theta}}$ = rotor hub pitching moment
 N = number of blades
 \hat{n} = unit vector normal to surface
 P_i = induced power
 Q = rotor shaft torque
 R = rotor radius
 r = blade radial station
 \mathbf{r} = vector from hub center to blade station r
 \hat{r} = unit vector along \mathbf{r}
 \bar{r}_c = nondimensional root cutout
 S = surface of control volume V
 T = rotor thrust
 U = velocity of blade normal to blade axis
 V = control volume
 \mathbf{V} = helicopter velocity vector
 v = induced velocity in wake

W = wake sheet surface
 w = induced velocity at rotor disk
 x = coordinate axis, positive aft
 Y = rotor in-plane side force
 y = coordinate axis, positive to right
 z = coordinate axis, positive up
 γ = circulation Lagrange multiplier
 $\boldsymbol{\gamma}$ = circulation Lagrange multiplier vector
 Γ = circulation
 $\boldsymbol{\Gamma}$ = vector of circulation at computational nodes
 θ_{tw} = linear blade twist
 κ = rotor-induced loss factor
 λ = vertical advance ratio, $V_z/\Omega R$
 μ = forward advance ratio, $-V_x/\Omega R$
 ν = vector of force Lagrange multipliers
 ν_T = thrust Lagrange multiplier
 ξ = Kelvin linear impulse
 Π = induced power functional
 ρ = density of air
 σ = rotor solidity, $Nc/\pi R$
 ϕ = potential function
 ψ = azimuthal position on rotor disk
 Ω = rotor rotational speed
 $\boldsymbol{\Omega}$ = rotor rotation vector, $\Omega \hat{k}$
 $\boldsymbol{\omega}$ = vector of moment Lagrange multipliers
 ω_x = rolling moment Lagrange multiplier
 ω_y = pitching moment Lagrange multiplier

Introduction

THE power required to drive the main rotor of a helicopter may be attributed to four sources: 1) profile power, 2) climb power, 3) parasite power, and 4) induced power. Of these, only the profile power and the induced power are directly affected by the design of the rotor. In this article, we consider how to achieve the minimum-induced power loss for a given forward flight condition by tailoring the lift distribution of the rotor.

It has long been recognized that tailoring the lift distribution of a rotor could improve performance. One method for changing the lift distribution is higher harmonic control (HHC) of blade pitch. Stewart¹ and Payne² considered the use of HHC to prevent the onset of blade stall on the retreating side, and compressibility losses on the advancing side. Stewart considered the use of 2/rev HHC. Payne extended this analysis to include both 2/rev and N/rev control. Stewart¹ concluded that by tailoring the lift distribution using 2/rev control, the max-

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